NAME:

QUIZ 10 FOR MATH 2551 F1-F4, NOVEMBER 28, 2018

This quiz should be taken without any notes and calculators. Time: 20 minutes. Show your work, otherwise credit cannot be given.

Problem 1: (3 points) Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$ where C is given by $x\vec{i} + (1-x^2)\vec{j}$ with x increasing from -1 to 1 and where $\vec{F} = 2x\vec{i} + 2y\vec{j}$.

$\vec{F}$ is a gradient field and hence the value depends only on the endpoints. Take the path $\vec{r}(x) = x\vec{i}$ then

$$\vec{F} \cdot \vec{r}'(x) = 2x$$

and integrating this function from -1 to 1 yields 0. Another way is to see that $\vec{F} = \nabla(x^2+y^2)$ and hence the value of the integral is the difference of the values of the function $x^2 + y^2$ at the points $(1, 0)$ and $(-1, 0)$ which is zero.

Problem 2: (3 points) Find the circulation of the field $\vec{F} = -y\vec{i} + x\vec{j}$ in the counterclockwise sense around the curve given by the two arcs $y = x^2$, $y = x$ in the first quadrant. (Hint: Use Green's theorem).

By Green's theorem we have $N_x - M_y = 2$. Hence the circulation is 2 times the area bounded by the curve.

$$\int_0^1 \int_{x^2}^x dydx = \int_0^1 x - x^2 dx = \frac{1}{6}$$

and hence the circulation is 1/3.

Problem 3: (4 points) Use the parametrization $\vec{r}(x, y) = x\vec{i} + y\vec{j} + \sqrt{x^2 + y^2}\vec{k}$ to compute the surface area of the cone given by $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 1$.

$$\vec{r}_x = \vec{i} + \frac{x}{\sqrt{x^2+y^2}}\vec{k}\ , \vec{r}_y = \vec{j} + \frac{y}{\sqrt{x^2+y^2}}\vec{k}$$

$$\vec{r}_x \times \vec{r}_y = \vec{k} - \frac{y}{\sqrt{x^2+y^2}}\vec{j} - \frac{x}{\sqrt{x^2+y^2}}\vec{i}$$

$$|\vec{r}_x \times \vec{r}_y| = \sqrt{2}$$

Integrating this over the disk $x^2 + y^2 \leq 1$ yields $\sqrt{2}\pi$