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QUIZ 6 FOR MATH 2551 F1-F4, OCTOBER 10, 2018

This quiz should be taken without any notes and calculators. Time: 20 minutes. Show your work, otherwise credit cannot be given.

Problem 1: (3 points) Find the points in the plane where the function $f(x,y) = x^2y + x - y$ has its local extrema.

$$f_x = 2xy + 1 = 0 \ , \ f_y = x^2 - 1$$

Hence $x = \pm 1$ and $y = -\frac{1}{2x} = \mp\frac{1}{2}$, i.e., the places for the local extrema are $(1, -\frac{1}{2})$ and $(-1, \frac{1}{2})$.

Problem 2: (4 points) The function $f(x,y) = 2x^2 - 4xy + y^2$ has $(0,0)$ as its only local extremum. What is its type, i.e., is it a local min, a local max or a saddle point?

We have

$$f_{xx} = 4 \ , \ f_{yy} = 2 \ , \ f_{xy} = -4 \ .$$

The determinant of the Hessian matrix is -8 and hence it is a saddle point.

Problem 3: (3 points) Use the method of Lagrange multipliers to find the point on the plane $3x + 2y + z = 6$ that is closest to the origin. (The right answer with any other method yields 1 point.)

We minimize the function $f(x,y,z) = x^2 + y^2 + z^2$ given that $g(x,y,z) = 3x + 2y + z - 6 = 0$.

$$\nabla f = 2\langle x, y, z\rangle \ , \ \nabla g = \langle 3, 2, 1\rangle$$

The Lagrange equation is $\nabla f = \lambda \nabla g$ so that

$$2x = 3\lambda \ , \ 2y = 2\lambda \ , \ 2z = \lambda$$

which yields

$$x = \frac{3\lambda}{2} \ , \ y = \lambda \ , \ z = \frac{\lambda}{2}$$

and the equation $3x + 2y + z - 6 = 0$ yields $\frac{9\lambda}{2} + 2\lambda + \frac{\lambda}{2} - 6 = 7\lambda - 6 = 0$ and hence

$$\lambda = \frac{6}{7} \ , x = \frac{9}{7} \ , \ y = \frac{6}{7} \ , \ z = \frac{3}{7}$$