

NAME:

QUIZ 8 FOR MATH 2551 F1-F4, OCTOBER 31, 2018

This quiz should be taken without any notes and calculators. Time: 20 minutes. Show your work, otherwise credit cannot be given.

Problem 1: (3 points) Change the following Cartesian integral into an equivalent polar integral and evaluate it.

$$\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2 + y^2) dx dy$$
$$\int_0^{\pi/2} \int_0^2 r^2 r dr = \frac{\pi r^4}{4} \Big|_0^2 = 2\pi$$

Problem 2: (4 points) Compute the area of the region bounded by the parabola $y = x - x^2$ and the line $y = -x$. (Hint: Sketch the region).

$$\int_0^2 \int_{-x}^{x-x^2} dy dx = \int_0^2 (2x - x^2) dx = \left[x^2 - \frac{x^3}{3} \right]_0^2 = \frac{4}{3}$$

Problem 3: (3 points) Use a triple integral to compute the volume of the tetrahedron cut from the first octant by the plane $2x + 2y + z = 2$.

$$\int_0^1 \int_0^{1-x} \int_0^{2-2x-2y} dz dy = 2 \int_0^1 \int_0^{1-x} (1-x-y) dy dx = 2 \int_0^1 \frac{(1-x)^2}{2} dx = \frac{1}{3}$$