NAME:

QUIZ 9 FOR MATH 2551 F1-F4, NOVEMBER 14, 2018

This quiz should be taken without any notes and calculators. Time: 20 minutes. Show your work, otherwise credit cannot be given.

Problem 1: (3 points) Find the volume of the solid enclosed by the cylinder $x^2 + y^2 = 4$, bounded above by the paraboloid $z = x^2+y^2$ and below by the xy-plane. (Hint: use cylindrical coordinates)

The integral is

$$\int_0^{2\pi} \int_0^2 \int_0^{r^2} dzrdrd\theta = 2\pi \int_0^2 r^3 dr = 8\pi$$

Problem 2: (3 points) Solve the system $u = x + y$, $v = x - y$ for x, y and compute the Jacobian $\frac{\partial(x,y)}{\partial(uv)}$.

$$x = \frac{u+v}{2} \ , y = \frac{u-v}{2}$$

and

$$\frac{\partial(x,y)}{\partial(uv)} = \left|\det \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}\right| = \frac{1}{2}$$

Problem 3: (4 points) Find the work done by the force $\vec{F} = \langle xy, y, -yz\rangle$ along the curve $\mathbf{r}(t) = (t, t^2, t)$, $0 \le t \le 1$ in the direction of increasing t.

$\vec{F}$ along the curve is given by

$$\langle t^3, t^2, -t^3\rangle$$

and

$$\mathbf{r}'(t) = \langle 1, 2t, 1\rangle$$

so that

$$\vec{F} \cdot \mathbf{r}'(t) = 2t^3$$

and the work is $\int_0^1 2t^3 dt = \frac{1}{2}$.