NAME:

SECTION:

TEST 1 FOR MATH 2551 F1-F4, SEPTEMBER 26, 2018

IMPORTANT: WRITE YOUE NAME AND SECTION NUMBER ON EVERY PAGE!

This test should be taken without any notes and calculators. Time: 50 minutes. Show your work and write legibly otherwise credit cannot be given. If you realize that you have written something which is wrong then, please, cross it out.

$$\vec{a} = \langle a_1, a_2, a_3 \rangle \ , \vec{b} = \langle b_1, b_2, b_3 \rangle$$
$$\vec{a} \times \vec{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$

Problem 1:

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Total:

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Problem 1: (10 points) Compute the area of the parallelogram spanned by the vectors

$$\langle 1,2,3\rangle \ , \langle 3,2,1\rangle \ .$$

(Check your answer!)

$$\langle 1,2,3\rangle \times \langle 3,2,1\rangle = \langle -4,8,-4\rangle$$

The area is given by the magnitude of the vector $\langle -4,8,-4\rangle = 4\langle -1,2,-1\rangle$ which is $4\sqrt{6}$.

Problem 2: Given the vector $\vec{v} = \langle 2,1,2\rangle$ and the vector $\vec{b} = \langle 1,2,1\rangle$.
a) (10 points) Find the projection $P_{\vec{v}}\vec{b}$ of the vector $\vec{b}$ onto the vector $\vec{v}$.

$$P_{\vec{v}}\vec{b} = \frac{\vec{v}\cdot\vec{b}}{|\vec{v}|^2}\vec{v} = \frac{6}{9}\langle 2,1,2\rangle = \frac{2}{3}\langle 2,1,2\rangle$$

b) (5 points) Compute the vector $\vec{b} - P_{\vec{v}}\vec{b}$.

$$\langle 1,2,1\rangle - \frac{2}{3}\langle 2,1,2\rangle = \frac{1}{3}\left[\langle 3,6,3\rangle - \langle 4,2,4\rangle\right] = \frac{1}{3}\langle -1,4,-1\rangle$$

c) (5 points) What can you say about the dot product of the vector $\vec{v}$ with $\vec{b} - P_{\vec{v}}\vec{b}$
This dot product must vanish!

Problem 3: Consider the point $(1, 2, 3)$ and the plane

$$x + y + z = 1 \ .$$

a) (5 points) Find any point P on the plane.

$$P = (1, 0, 0)$$

b) (5 points) Find the distance vector between the point P you found in problem a) and the given point $(1, 2, 3)$.

$$\langle 0, 2, 3 \rangle$$

c) (10 points) Compute the distance of the point $(1, 2, 3)$ to the plane $x + y + z = 1$.
Project the vector obtained in problem b) onto the vector normal to the plane, which is given by $\langle 1, 1, 1 \rangle$. This is

$$\frac{5}{3}\langle 1, 1, 1 \rangle$$

and the distance is

$$\frac{5}{\sqrt{3}}$$

Problem 4: Given the Helix

$$\vec{r}(t) = \langle \cos t, \sin t, t \rangle \ , t \in \mathbb{R} \ .$$

a) (5 points) Find the velocity vector $\vec{v}(t)$.

$$\langle -\sin t, \cos t, 1 \rangle$$

b) (10 points) Find the line tangent to the Helix at the point given by $t = \pi/2$.
The velocity vector at that point is $\langle -1, 0, 1 \rangle$. The point common to the Helix and the tangent line is $(0, 1, \pi/2)$. Hence the tangent line is given by

$$(0, 1, \pi/2) + s\langle -1, 0, 1 \rangle$$

or

$$x = -s, \ y = 1, \ z = \pi/2 + s$$

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Problem 5: A particle has a trajectory given by $\vec{r}(t) = \langle \cos t, \sin t, -5t^2 \rangle$
a) (5 points) Find the speed $s'(t)$ of the particle.

The velocity vector is $\vec{v}(t) = \langle -\sin t, \cos t, -10t \rangle$ and the speed $s'(t) = \sqrt{1 + 100t^2}$

b) (5 points) Find the tangential acceleration a_T.

$$a_T = s''(t) = \frac{100t}{\sqrt{1 + 100t^2}}$$

c) (10 points) Find the normal acceleration a_N.

The acceleration is

$$\vec{a}(t) = \langle -\cos t, -\sin t, -10 \rangle$$

so that $|\vec{a}|^2 = 1 + 100 = 101$. Now

$$a_N^2 = |\vec{a}|^2 - a_T^2 = 101 - \frac{10000t^2}{1 + 100t^2} = \frac{101 + 10100t^2 - 10000t^2}{1 + 100t^2} = \frac{101 + 100t^2}{1 + 100t^2}$$

Hence

$$a_N = \sqrt{\frac{101 + 100t^2}{1 + 100t^2}}$$

Problem 6: True or false (no partial credit).

a) (5 points) The function

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

is not continuous at $(0, 0)$. TRUE

b) (5 points) If $f(x, y)$ is a continuous function on $\mathbb{R}^2$ and $g(s)$ is a continuous function on $\mathbb{R}$, then $g(f(x, y))$ is a continuous function on $\mathbb{R}^2$. TRUE

c) (5 points) If $f(x, y)$ converges to $f(0, 0)$ along every straight line through the origin, then $f(x, y)$ is continuous at $(0, 0)$. FALSE