HOMEWORK 7

Problem 1: Suppose that V, W are two vector spaces and $T : V \to W$ is a linear transformation. The transformation T is called **injective** if $T(\vec{v}_1) = T(\vec{v}_2)$ implies that $\vec{v}_1 = \vec{v}_2$. The transformation T is called **surjective** if for any $\vec{w} \in W$ there exists $\vec{v} \in V$ such that $T(\vec{v}) = \vec{w}$.

a) Show that T is injective if and only if $Ker(T) = \{\vec{0}\}$. Recall that $Ker(T)$, the **kernel** of T consists of all vectors $\vec{v} \in V$ such that $T(\vec{v}) = 0$.

b) Show that T is surjective if and only if the **range** of T, $Ran(T) = W$. Recall that $Ran(T)$ is the set of all vetors in $\vec{w} \in W$ that are of the form $\vec{w} = T(\vec{v})$ for some $\vec{v} \in V$.

c) Show that if T is injective and surjective then it has an inverse $T^{-1} : W \to V$.

d) Show that this inverse T^{-1} is linear.

Please work problems 12, 34 and 35 in Section 4.2 of Strang. Work also problems 5 and 11 in Section 4.3 of Strang.

Please turn it in for grading on Thursday March 5 .