PRACTICE TEST 1

Problem 1: a) By computing the row reduced echelon form find all the solutions of the system $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$$

b) Indicate the pivot columns.
c) What is the rank of A?

Problem 2: a) Find a 3×3 matrix E that when multiplied with a 3×3 matrix A adds three times the first *column* of A to the second *column* of A. (Hint: Think of AE and not EA.)
b) What is the inverse of E?

Problem 3: a) Are the three vectors below linear independent?

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

b) An $n \times n$ matrix is invertible if and only if the column vectors form a basis for $\mathbb{R}^n$. Explain this.

Problem 4: Find the inverse of the matrix

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

Problem 5: a) Find a 2×2 matrix whose column space and null space are equal.
b) Is the same true for a symmetric 2×2 matrix? Explain.

Problem 6: Find a basis for $N(A), C(A), N(A^T)$ and $C(A^T)$ where

$$A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 6 & 6 \\ 2 & 3 & 0 \end{bmatrix}$$

Problem 7: Using the normal equations, solve the least square problem for $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 2 & -1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$$

Problem 8: a) Find the QR factorization of the matrix

$$A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 6 & 6 \\ 2 & 3 & 0 \end{bmatrix}$$

b) Using the result of a) find the least square solutions for the equation $A\vec{x}$ where $\vec{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

Problem 9: Explain why it is true or provide a counter example.

a) The column space does not change under row reduction.

b) A matrix has full column rank if and only if it has a trivial null space.

c) For any matrix A, $N(A^TA)$ and $C(A^TA)$ are perpendicular.

d) If A is an $m \times n$ matrix, then $C(A^TA) = C(A^T)$.

e) Three vectors of which any two vectors are linearly independent are linearly independent.